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Signals from extra dimensions decoupled from the compactification scale

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Abstract

Multilocalization provides a simple way of decoupling the mass scale of new physics from the compactification scale of extra dimensions. It naturally appears, for example, when localization of fermion zero modes is used to explain the observed fermion spectrum, leaving low energy remnants of the geometrical origin of the fermion mass hierarchy. We study the phenomenology of the simplest five dimensional model with order one Yukawa couplings reproducing the standard fermion masses and mixing angles and with a light Kaluza-Klein quark $Q_{2/3}$ saturating experimental limits on V_{tb} and m_Q, and then with observable new effects at large colliders.

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1 Introduction

Theories with extra dimensions have received a great deal of attention during the last years [1, 2, 3]. They not only help to explain four dimensional puzzles but predict new physics at observable scales. Thus, the hierarchy of fermion masses and mixing angles can be related to the localization of the corresponding zero modes around different points in the extra dimensions [4]. These models reproduce the standard four dimensional fermion spectrum with order one Yukawa couplings, the small mass ratios and mixing angles resulting from the small overlapping of the corresponding wave functions in the extra dimensions [4, 5, 6, 7] [3]. However, the fermion splitting can induce large flavour changing neutral currents (FCNC) mediated by the Kaluza-Klein (KK) tower of gauge bosons [9], and then the experimental constraints on rare processes involving the first two families banish the masses of the first excited gauge bosons and the compactification scale M_c to high energy. But as we will show, multilocalization can still leave remnants at low energy.

It has been proven in five dimensional models with localized gravity that the addition of mass terms modifies the localization properties of the fields, with the multilocalization of the first KK modes resulting in light four dimensional masses decoupled from the effective compactification scale [10]. This phenomenon, however, is general and independent of the gravitational background. As a matter of fact, it is a usual companion of nontrivial mass terms, and then can naturally appear in flavour models in extra dimensions.

In Section 2 we study the simplest five dimensional model with multilocalized fermions. We assume a flat extra dimension compactified on $\frac{S^1}{Z_2}$ and add step function masses, which can be thought as a limit of more realistic scalar backgrounds. In fact, the corresponding four dimensional effective lagrangian is the same as for more elaborate theories [4]. In an Appendix we show that in models with split fermions compactified on $\frac{S^1}{Z_2}$ (like, for instance, the one recently proposed in [12]) there is also in general multilocalization, and then massless chiral fermions and light KK modes decoupled from M_c. We use these results in Section 3 to construct a definite model with order one Yukawa couplings reproducing the standard quark masses and mixing angles and with an additional vector-like quark of charge $\frac{2}{3}$, $Q_{2/3}$, near the electroweak scale saturating the experimental limit on V_{tb}, and then observable in forthcoming experiments. Finally Section 4 is devoted to phenomenological implications and conclusions.

[3] For another approach to the flavour problem in extra dimensions see [8].

[4] This also applies to deconstructing models in four dimensions [11].

2 Light Kaluza-Klein fermions in flat space

The appearance of new states parametrically lighter than the compactification scale is a consequence of multilocalization. It has been recently discussed in great detail in the case of warped compactifications for particles with spin smaller than two [10] (for the graviton it was first studied in [13] and further developed in [14].) It can be also present, however, in a flat background. We review in this Section fermion multilocalization in the simplest possible context, a five dimensional model in flat space with the extra dimension compactified on the orbifold $\frac{S^1}{Z_2}$. This is a circle of radius R with the Z_2 identification $y \to -y$ or an interval $0 \le y \le \pi R$ with two boundaries, the orbifold fixed points.

The action of a spinor reads in this space (we use the "mostly minus" convention for the metric $(+,-,-,-,-)$ and $\gamma^4 = \mathrm{i}\gamma^5$)

$$S = \int \mathrm{d}^4x \int_0^{\pi R} \mathrm{d}y \, \bar{\Psi}[\mathrm{i}\partial_N \gamma^N - M(y)]\Psi, \quad N = 0, \ldots, 4, \tag{1}$$

where the Dirac mass, which is odd under the action of Z_2, can be chosen to have a multi-kink structure in order to provide the desired multilocalization

$$M(y) = \begin{cases} M, & 0 \le y \le \pi a, \\ -M, & \pi a \le y \le \pi R, \end{cases} \tag{2}$$

with $0 \le a \le R$. Note that the mass M has to be real by hermiticity but there is no restriction on its sign. The kink-antikink shape for the mass term [15] is recovered in the limit $a \to R$.

The KK reduction is performed in the usual way. We split the five dimensional vector-like fermion into its two chiralities $\Psi = \Psi_L + \Psi_R$ satisfying $\gamma^5 \Psi_{L,R} = \mp \Psi_{L,R}$, and expand in KK modes

$$\Psi_{L,R}(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} f_n^{L,R}(y) \Psi_{L,R}^{(n)}(x). \tag{3}$$

Substituting them into the action, the decoupling of the quadratic terms follows from the relations

$$\int_0^{\pi R} \mathrm{d}y \, \frac{f_n^L f_m^L}{\pi R} = \int_0^{\pi R} \mathrm{d}y \, \frac{f_n^R f_m^R}{\pi R} = \delta_{nm}, \tag{4}$$

$$(\partial_y - M(y)) f_n^L = -m_n f_n^R \quad , \quad (-\partial_y - M(y)) f_n^R = -m_n f_n^L. \tag{5}$$

Then the corresponding four dimensional lagrangian describes a chiral zero mode plus a tower of vector-like fermions.

The action of Z_2 on Ψ can be chosen to be $\Psi(-y) = \gamma^5 \Psi(y)$ or $-\gamma^5 \Psi(y)$. For the shake of concreteness we take the first choice, implying that the Right Handed (RH) component is even $\Psi_R(-y) = \Psi_R(y)$ and the Left Handed (LH) one odd $\Psi_L(-y) = -\Psi_L(y)$. The other assignment can be obtained from our results by just interchanging Ψ_L and Ψ_R and replacing M by $-M$. The coupled first order differential equations (5) imply the second order one

$$(-\partial_y^2 - M'(y) + M^2) f_n^R = m_n^2 f_n^R, \tag{6}$$

where the prime stands for ∂_y and

$$M'(y) = 2M[\delta(y) - \delta(y - \pi a) + \delta(y - \pi R)]. \tag{7}$$

f_n^L, which satisfies the same equation but with $M \to -M$, can be also obtained from f_n^R for $n \neq 0$ using the second equation in (5):

$$f_n^L = \frac{1}{m_n}(\partial_y + M(y)) f_n^R. \tag{8}$$

The solution of the Schrödinger equation (6) for f_n^R is obtained imposing the corresponding boundary conditions at $y = 0, \pi a$ and πR,

$$\begin{aligned}
f_n^{R\,\prime}(0) &= -M f_n^R(0), && (9)\\
f_n^R(\pi a - \epsilon) &= f_n^R(\pi a + \epsilon), && (10)\\
f_n^{R\,\prime}(\pi a + \epsilon) - f_n^{R\,\prime}(\pi a - \epsilon) &= 2M f_n^R(\pi a), && (11)\\
f_n^{R\,\prime}(\pi R) &= M f_n^R(\pi R), && (12)
\end{aligned}$$

where the limit $\epsilon \to 0$ is understood. The Z_2 projection leaves a chiral zero mode with even chirality, *i.e.* f_0^R. The odd boundary conditions, which imply the vanishing of the wave function at the orbifold fixed points, are not compatible with a massless mode. The zero mode wave function reads

$$f_0^{\mathrm{R}}(y) = \begin{cases} A \exp[-M(y - \pi a)], & 0 \leq y \leq \pi a, \\ A \exp[M(y - \pi a)], & \pi a \leq y \leq \pi R, \end{cases} \tag{13}$$

where $A = \left[\frac{2M\pi R}{\exp[2M\pi a] + \exp[2M\pi(R-a)] - 2}\right]^{1/2}$ is a normalization constant. Thus it is exponentially localized at both orbifold fixed points for $M > 0$ and at $y = \pi a$ (intermediate brane) for $M < 0$. (The opposite would happen in the case of an even LH zero mode.)

All the other KK fermions are vector-like, with the first massive mode $f_1^{L,R}$ having distinctive properties in the case of multilocalization. This occurs if the parameters in the potential, Eqs. (6,7), satisfy

$$2M\pi a(R - a) > R. \tag{14}$$

Obviously, the case $a = R$ does not fulfil Eq. (14) and there is no multilocalization for any value of the mass parameter. The phenomenology of this case [6, 7, 12, 16], which is also quite interesting, can be obtained from our results taking the continuous limit $a \to R$.

In the case of multilocalization the first excited state is also exponentially localized

$$f_1^R(y) = \left\{ \begin{array}{ll} B\left(\mathrm{e}^{k_1 y} + \frac{k_1+M}{k_1-M}\mathrm{e}^{-k_1 y}\right), & 0 \le y \le \pi a, \\ C\left(\mathrm{e}^{-k_1(y-\pi R)} + \frac{k_1+M}{k_1-M}\mathrm{e}^{k_1(y-\pi R)}\right), & \pi a \le y \le \pi R, \end{array} \right. \tag{15}$$

and

$$f_1^L(y) = \left\{ \begin{array}{ll} D\left(\mathrm{e}^{k_1 y} - \mathrm{e}^{-k_1 y}\right), & 0 \le y \le \pi a, \\ E\left(\mathrm{e}^{-k_1(y-\pi R)} - \mathrm{e}^{k_1(y-\pi R)}\right), & \pi a \le y \le \pi R, \end{array} \right. \tag{16}$$

with B (D) and C (E) constants fixed by normalization and continuity conditions. Its mass $m_1^2 = M^2 - k_1^2$, with k_1 the solution of the eigenvalue equation

$$k - M - (k+M)\mathrm{e}^{-2k\pi R} + M\left[\mathrm{e}^{-2k\pi a} + \mathrm{e}^{-2k\pi(R-a)}\right] = 0, \tag{17}$$

can be $\ll M^2$. Eq. (17) has always only one solution with $0 < k_1 < M$. The extreme case $m_1 \to 0$ ($m_1 \to M$) corresponds to $2M\pi a(R-a) \gg R$ ($2M\pi a(R-a) \sim R$).

The rest of the spectrum consists of oscillating states heavier than M and with a spacing of order $\sim \frac{1}{R}$. In Fig. 1 we show the masses of the first KK fermions as function of the five dimensional mass M in units of $M_c = \frac{1}{R}$ (we have fixed $a = R/2$ for illustration). As can be observed, there is only one light KK vector-like fermion f_1 multilocalized for $MR > 2/\pi$, see Eq. (14).

In the absence of FCNC precision data typically require $M_c \gtrsim 4$ TeV, otherwise this limit can be as large as ~ 5000 TeV [9, 17]. This makes a priori the effects of the heavy states small compared to the contribution of the multilocalized one.

As has been recently discussed in [10] the appearance of multilocalization and the related light KK modes can be easily understood in terms of the shape of the potential in the corresponding Schrödinger equation. We review here the argument for completeness. It has been known for a long time that domain wall backgrounds create potential wells which can confine massless fermion zero modes in their world volume [18]. In the case that the potential has a double well shape with each well supporting a bound state (multidomain wall background), the resulting spectrum consists on two nearly degenerate modes with a mass splitting proportional to the quantum tunneling probability between wells. In the limit of large separation, the lightest state (which is always massless) corresponds to the even combination of the bound states for the separate wells, while the odd combination has a mass proportional to the quantum tunneling probability. The absolute values of the corresponding wave functions only differ in the intermediate region where they are exponentially suppressed, what results in the exponentially small mass for the first excited mode. In Fig. 2 we draw the potential for the two signs of M in Eq. (7): the case $M < 0$ (left) corresponds to the left region in Fig. 1 with no multilocalization, and the case $M > 0$ (right) to the right region in the same Figure with the double well shape of the potential

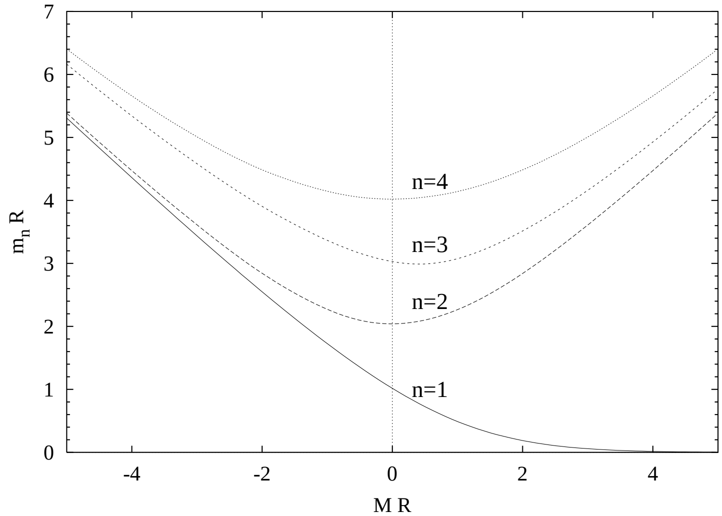

Figure 1: Values of the masses of the first KK modes as a function of the five dimensional mass in units of $1/R$. We have fixed $a = R/2$.

producing multilocalization for M large enough. The wave functions of the zero mode and the first KK excitation are shown in Fig. 3. Again the left (right) plot corresponds to the left (right) region in Fig. 1. It can be observed that in the latter case the difference between the absolute values of the wave functions are exponentially suppressed.

In our case the potential consists of delta function wells and barriers because of the step function mass term we have considered. A possibly more realistic background would consist of hyperbolic-tangent shaped masses [15], with step functions being considered as a (thin brane) limit. From the discussion above it should be clear that in this more realistic case, provided that the scalar background has a three domain wall shape, multilocalization can be also present. As an example, we show in the Appendix that the alternative limit with the intermediate brane becoming fat and the boundary branes thin (which has been considered recently in [12], realizing the split fermion idea on the orbifold $\frac{S^1}{Z_2}$) also presents multilocalization. Finally we would like to mention the importance that the orbifold has in our construction. As has been emphasized, multilocalization will generically occur provided that the scalar background giving mass to the five dimensional fermions has a multidomain wall structure. Orbifold models, which allow to obtain a chiral spectrum in five dimensions, naturally induce domain walls at both orbifold fixed points [15]. Thus, any scalar potential with an intermediate domain wall solution in the orbifold (what can be accomplished by the appropriate boundary sources [12]) automatically presents the multikink structure leading to multilocalization.

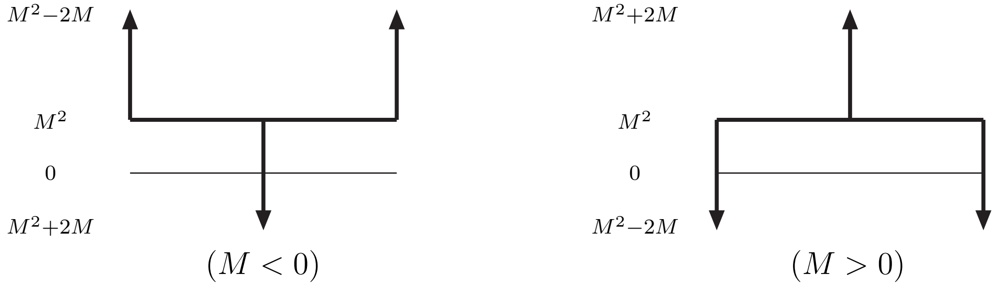

Figure 2: Potential $M^2 - M'(y)$ of the equivalent Schrödinger equation for a multikink mass term in arbitrary units (for $a = R/2$). On the left there is no multilocalization ($MR < 2/\pi$), in contrast with the potential on the right which does multilocalize ($MR > 2/\pi$).

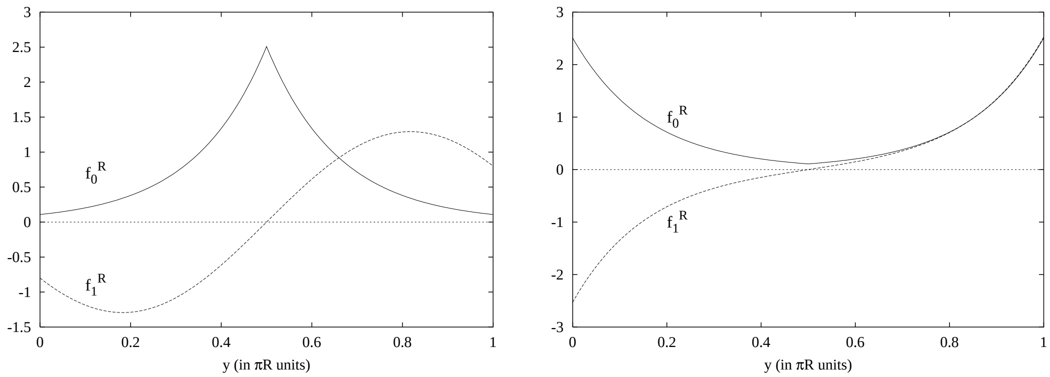

Figure 3: Profiles of the massless zero mode f_0^R and the first KK excitation f_1^R with no multilocalization, $MR = -2$ (left), and with multilocalization, $MR = 2$ (right).

3 A model of flavour with light Kaluza-Klein quarks

In this Section we construct a five dimensional model compactified on $\frac{S^1}{Z_2}$ with the standard fermion content, three $SU(2)_L \times U(1)_Y$ doublets q_i with LH components even and six singlets u_i, d_i with even RH components, which reduces below M_c to the Standard Model (SM) plus a light vector-like quark of charge 2/3 with observable mixing with the top quark. The mass terms, $M_i^{q,u,d}$, are order the compactification scale with step function shape and the Yukawa couplings, $\lambda_{ij}^{u,d(5)}$, order one. We shall make a detailed numerical discussion to show that there is no apparent fine tuning, and to emphasize its phenomenological relevance. Although the model is idealized, the four dimensional lagrangian is the same as for more realistic cases as already emphasized.

The five dimensional action reads

$$S = \int \mathrm{d}^4x \int_0^{\pi R} \mathrm{d}y \; \Big\{ \bar{q}_i[\mathrm{i}\gamma^N D_N - M_i^q(y)]q_i + (\mathrm{q} \to \mathrm{u}, \mathrm{d})$$

$$+ \delta(y)[\lambda_{ij}^{u(5)}\bar{q}_i u_j \tilde{\phi} + \lambda_{ij}^{d(5)}\bar{q}_i d_j \phi + \text{h.c.}]\Big\}\,, \tag{18}$$

where D_N is the covariant derivative and to maintain the discussion simple we assume that the Higgs ϕ is at a fixed point, to be concrete at $y = 0$. We also assume that all the five dimensional masses M are generated from a unique scalar background and therefore that a is common to all of them. Performing the KK decomposition and integrating the extra dimension one obtains the four dimensional lagrangian

$$\begin{aligned} \mathcal{L} &= \sum_{n=0}^{\infty}\Big\{\bar{q}_i^{(n)}[\mathrm{i}\gamma^\mu D_\mu - m_n^{q_i}]q_i^{(n)} + (\mathrm{q}\to\mathrm{u},\mathrm{d})\Big\} \\ &+ \sum_{n,m=0}^{\infty}\Big\{\lambda_{ij}^{u(nm)}\bar{q}_i^{(n)}u_j^{(m)}\tilde{\phi} + \lambda_{ij}^{d(nm)}\bar{q}_i^{(n)}d_j^{(m)}\phi + \text{h.c.}\Big\}\,, \end{aligned} \tag{19}$$

where D_μ only includes the gauge boson zero modes and $m_0 = 0$. The effective four dimensional Yukawa couplings are

$$\lambda_{ij}^{u,d(nm)} = \frac{\lambda_{ij}^{u,d(5)}}{\pi R} f_n^{q_i} f_m^{u_j,d_j}, \tag{20}$$

where the wave functions f are evaluated at $y = 0$. For five dimensional masses $M \neq 0$ the exponential localization of the fermion zero modes can easily give the observed hierarchy of Yukawa couplings [5]

$$\lambda_{ij}^{u,d(00)} \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \tag{21}$$

if $f_0^{q_i} \sim f_0^{u_i} \sim f_0^{d_i} \sim (\epsilon^2, \epsilon, 1)$. This matrix has two zero eigenvalues which become order ϵ^2 and ϵ^4 when the order one five dimensional Yukawa couplings are also included. To obtain such a hierarchy the third family must be strongly localized near the Higgs boundary and the first two families at $y = \pi a$, so that they are suppressed at the fixed point where the Higgs lives. As shown in Fig. 1 for M large enough (and $0 < a < R$) there is also multilocalization and a light KK vector-like fermion. In this scenario it is natural to have t_R more strongly localized than t_L since the localization of the latter is the same as the one of its doublet counterpart b_L, which has a smaller Yukawa. This and the fact that with only one light vector-like quark we can account for large deviations of the SM top couplings make sufficient to have only t_R multilocalized.

The effective lagrangian for the three light families is obtained integrating out the tower of KK modes in Eq.(19). This has been done in detail in [16]. The corrections due to the KK fermions are proportional to the masses of the corresponding zero modes and to

[5] A similar texture for the Yukawa matrices is obtained in [19] (see also [20] for family symmetries giving the same mass matrices). The order of the different entries can be slightly changed varying the five dimensional Yukawa couplings.

the inverse of the KK masses squared. Therefore only the top has large corrections and the main contribution is from the light multilocalized KK state. Indeed the largest top couplings to Z and $W^\pm$ in the mass eigenstate basis, $X_{tt}^{L,R}$ [6] and W_{tb}^L, respectively, (W_{tb}^R has an extra suppression factor m_b/m_t) are to first order in $1/m_n^2$ [16]

$$
\begin{aligned}
X_{tt}^L &= 1 - m_t^2 \sum_{k=1}^{3} |(U_R^u)_{kt}|^2 \sum_{n=1}^{\infty} \left(\frac{f_n^{u_k}}{m_n^{u_k} f_0^{u_k}} \right)^2 , &(22)\\
X_{tt}^R &= m_t^2 \sum_{l,k,r=1}^{3} V_{tl} (U_L^q)^\dagger_{lk} \sum_{n=1}^{\infty} \left(\frac{f_n^{q_k}}{m_n^{q_k} f_0^{q_k}} \right)^2 (U_L^q)_{kr} V_{rt}^\dagger , &(23)\\
W_{tb}^L &= V_{tb} - \frac{1}{2} m_t^2 \sum_{k=1}^{3} |(U_R^u)_{kt}|^2 \sum_{n=1}^{\infty} \left(\frac{f_n^{u_k}}{m_n^{u_k} f_0^{u_k}} \right)^2 , &(24)
\end{aligned}
$$

where we have neglected corrections suppressed by $\frac{m_i V_{ib}}{m_t V_{tb}}$ with $i = u, c$, and the unitary matrices U diagonalize

$$
\begin{aligned}
(U_L^q)^\dagger_{ik} \lambda_{kl}^{u(00)} \frac{v}{\sqrt{2}} (U_R^u)_{lj} &= V_{ij}^\dagger m_j^u, \quad m_{1,2,3}^u = m_{u,c,t} &(25)\\
(U_L^q)^\dagger_{ik} \lambda_{kl}^{d(00)} \frac{v}{\sqrt{2}} (U_R^u)_{lj} &= m_j^d, \quad m_{1,2,3}^d = m_{d,s,b} &(26)
\end{aligned}
$$

where v is the Higgs vev and, in the absence of KK corrections, $m_j^{u,d}$ are the mass eigenvalues and the unitary matrix V is the experimentally measured CKM matrix. Whereas the corrected top mass

$$
m_t^{\rm phys} = m_t \left(1 - \frac{1}{2} m_t^2 \sum_{k=1}^{3} |(U_R^u)_{kt}|^2 \sum_{n=1}^{\infty} \left(\frac{f_n^{u_k}}{m_n^{u_k} f_0^{u_k}} \right)^2 \right) . \tag{27}
$$

Multilocalization allows for a geometrical explanation of the hierarchy of fermion masses and mixings and at the same time for the existence of exotic fermions at observable energies. Let us work out a numerical example along the lines discussed above. Fixing, for example, $a = R/2$ [7] and $R = (85 \text{ TeV})^{-1}$ with

$$
\begin{aligned}
M_i^q R &= (4.02, 2.23, -0.50),\\
M_i^u R &= (-2.68, 0.57, 4.85),\\
M_i^d R &= (-3.43, -3.02, -2.01),
\end{aligned} \tag{28}
$$

and

$$
\frac{\lambda_{ij}^{u(5)}}{\pi R} = \begin{pmatrix} 0.21 & -0.36 & 0.11 \\ -0.11 & 0.06 & -0.08 \\ 0.29 & 0.16 & 0.18 \end{pmatrix}, \quad \frac{\lambda_{ij}^{d(5)}}{\pi R} = \begin{pmatrix} 0.35 & 0.10 & 0.46 \text{ e}^{-2.77\text{ i}} \\ -0.17 & -0.65 & 0.10 \\ 0.25 & 0.13 & 0.25 \end{pmatrix}, \tag{29}
$$

[6] They do not include the electromagnetic component, being equal to 1 and 0 in the SM, respectively.

[7] In this case the same model can be obtained with kink-antikink masses compactifying on $\frac{S^1}{Z_2 \times Z_2'}$ [21].

we obtain the observed masses and mixing angles for the known quarks plus a vector-like quark of charge 2/3 with a mass $m_1^{u_3} = 400$ GeV. The other KK fermions have masses $m_n \gtrsim 60$ TeV. The observed CP violation is related to five dimensional Yukawa couplings which are in general complex, being however enough that only $\lambda_{13}^{d(5)}$ is complex to reproduce the measured CP violation. Integrating the KK modes [16], Eqs.(22-27), we find (in GeV)

$$m_{u,c,t}^{\rm phys} = (6 \times 10^{-3}, 2.7, 165), \quad m_{d,s,b}^{\rm phys} = (11 \times 10^{-3}, 0.26, 6.6), \tag{30}$$

where these masses are to be compared with the running $\overline{\rm MS}$ masses evaluated at the scale $m_t^{\rm phys}$. We have used for the masses the experimental values in [22], taking into account the running up to the scale $m_t^{\rm phys}$ [23],

$$m_u^{\rm phys} = 2.9 - 9.2 \text{ MeV} \quad , \quad m_d^{\rm phys} = 5.5 - 16.5 \text{ MeV}, \tag{31}$$
$$m_c^{\rm phys} = 2.5 - 2.9 \text{ GeV} \quad , \quad m_s^{\rm phys} = 140 - 310 \text{ MeV}, \tag{32}$$
$$m_t^{\rm phys} = 161 - 171 \text{ GeV} \quad , \quad m_b^{\rm phys} = 6.2 - 6.8 \text{ GeV}. \tag{33}$$

Whereas the corrected CKM matrix writes in the PDG phase convention [22]

$$W^L = \begin{pmatrix} 0.9748 & 0.223 & 0.003\ \mathrm{e}^{-1.05\ \mathrm{i}} \\ -0.223 - 0.00012\ \mathrm{e}^{1.05\ \mathrm{i}} & 0.9739 - 0.00006\ \mathrm{e}^{1.05\ \mathrm{i}} & 0.040 \\ 0.008 - 0.003\ \mathrm{e}^{1.05\ \mathrm{i}} & -0.036 - 0.0007\ \mathrm{e}^{1.05\ \mathrm{i}} & 0.9045 \end{pmatrix}. \tag{34}$$

The amount of CP violation can be also given using the Jarlskog invariant [24]

$$\mathrm{Im}(W_{ub}^L W_{cs}^L W_{us}^{L*} W_{cb}^{L*}) = 2.4 \times 10^{-5}, \tag{35}$$

or the β angle [22]

$$\sin(2\beta) = \sin\left[2\arg\left(-\frac{W_{cd}^L W_{cb}^{L*}}{W_{td}^L W_{tb}^{L*}}\right)\right] = 0.65, \tag{36}$$

which is in agreement with the most recent BaBar [25] and Belle [26] measurements

$$\sin(2\beta) = 0.59 \pm 0.14 \pm 0.05 \quad \text{and} \quad 0.99 \pm 0.14 \pm 0.06, \tag{37}$$

respectively. As can be observed, the masses and mixing angles in Eqs. (30,34) give the correct experimental values, differing appreciably from the minimal SM only

$$W_{tb}^L = 0.90, \tag{38}$$

and

$$X_{tt}^L = 0.81. \tag{39}$$

This reflects the lack of unitarity of the CKM matrix which is in turn due to the top mixing with the lightest vector-like KK fermion, which is predicted to have a mass

$$m_Q = 440 \text{ GeV}. \tag{40}$$

Several comments are in order. First, the required minimization to fit the experimental values fixing the masses and Yukawas in Eqs. (28,29) is essentially determined by the form of the four dimensional couplings in Eq. (20) and their approximate geometrical form, Eq. (21). Second, this structure is induced by the five dimensional masses through the (multi)localization of the zero modes, see for example Fig. 4. The quarks can be pushed to the four dimensional boundary taking the corresponding wave functions $f_{1,2,\ldots} \to 0$ and $f_0 \to \sqrt{\pi R}$. Third, the fit has been done demanding a large top mixing, this being related to the mass of the light vector-like quark. However, this correlation is lost if we allow to vary a and MR independently. In Fig. 5 we plot for arbitrary W_{tb}^L and m_Q the curves with a and $M_3^u R$ fixed, respectively, our model corresponding to $a = R/2$ and $M_3^u R = 4.85$. Finally, we want to emphasize that all our results are obtained at tree level. Large departures from the SM require a reanalysis of radiative corrections.

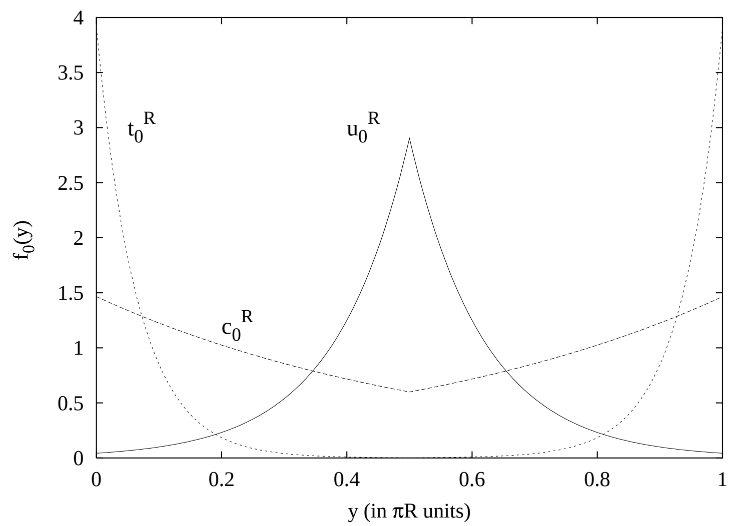

Figure 4: Profiles of the RH up quark zero modes for the five dimensional masses given in the text. Only t_R is multilocalized.

4 Phenomenological implications and conclusions

We have discussed fermion multilocalization in a five dimensional model with the flat extra dimension compactified on $\frac{S^1}{Z_2}$ and step function masses. This phenomenon has been studied for particles up to spin 2 in warped backgrounds [10], but also happens in other backgrounds in the presence of domain wall masses. Multilocalization allows for a geometrical interpretation of the hierarchy of fermion masses and mixing angles and at the same time for the decoupling of vector-like fermions from the compactification scale, making

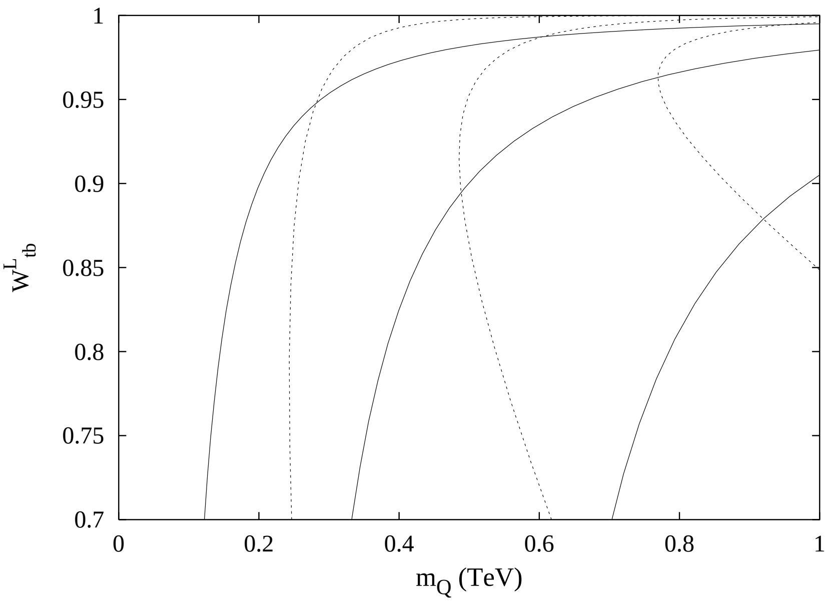

Figure 5: Top coupling W_{tb}^L and lightest vector-like quark mass m_Q as a function of the intermediate brane position a and the five dimensional mass M_3^u. Solid (dashed) lines correspond to fixed $\frac{a}{R}$ ($M_3^u R$) values, from left to right $0.525, 0.5, 0.475$ ($5.55, 4.85, 4.5$). We take as in the text $R = (85 \quad \text{TeV})^{-1}$.

them observable.

In models with localized fermions at separate points in the extra dimensions there are a priori large FCNC, requiring compactification scales up to $M_c \gtrsim 5000$ TeV [9]. This bound is typically reduced for split fermion models with kink-antikink masses to $M_c \gtrsim 230$ TeV in the case of a boundary Higgs [12]. If these limits are not evaded by the specific model, and all the KK excitations have masses order M_c or higher, no signal of the extra dimensions is expected at future colliders. However, this is not the case if there is multilocalization because the lightest KK modes decouple from M_c. The specific model we have worked out can accommodate a new vector-like quark of charge $2/3$ with a mass $m_Q = 440$ GeV and a slightly lower compactification scale $M_c = 85$ TeV, evading all FCNC constraints.

Exotic quarks near the electroweak scale have been extensively studied in the past [27]. They are present in many grand unified models, like for instance E_6. Models in extra dimensions with the new vector-like quarks being the KK excitations of bulk fermions provide a natural realization of this possibility. In general the KK towers of vector-like fermions manifest at low energy modifying the fermion mixing [28], with the deviations from the SM predictions scaling with the masses of the SM quarks involved [16]. This which may be also indicated by experiment is naturally realized in these models, in contrast with former unified scenarios, what together with present experimental limits make the top, and then large colliders, the best place to look for these new effects. In the specific case we have considered Q can be directly observed, the reach of Tevatron being several hundreds GeV and of LHC few TeV [16, 29]. This vector-like quark also mixes with the top, modifying its couplings $W_{tb}^L = 0.90$ and $X_{tt}^L = 0.81$. Such a departure of the SM predictions, $W_{tb}^L \lesssim X_{tt}^L = 1$, can be established for W_{tb}^L at Tevatron Run II with an accumulated luminosity of 30 fb^{-1}, the expected accuracy being 7.6% [30]; and similarly at the LHC but with an expected precision of 5% [31]. Whereas X_{tt}^L will be measured with a precision of 2% at TESLA [32, 33]. In Fig. 5 we see that this model can fit any value of W_{tb}^L and m_Q varying the intermediate brane position and the five dimensional step function mass for a given compactification scale. We have neglected any consideration related to the size of the radiative corrections.

As a final comment we would like to mention that we have concentrated on the quark sector, but multilocalization can be also used for leptons. A first attempt was made in [34] (see also [7]).

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A Split fermion multilocalization

In this Appendix we show how multilocalization naturally appears in five dimensional models with split fermions and the flat extra dimension compactified on $\frac{S^1}{Z_2}$ [12]. Fermions are allowed to live in the bulk and have a linear odd mass term

$$M(y) = -\frac{M}{R}(y - \pi a), \tag{41}$$

with $0 \leq a \leq R$. A mass term of this kind can be obtained from a bulk scalar with appropriate source terms at the orbifold fixed points plus a step function mass term for $a \neq R/2$ [12]. For simplicity, we will consider the case $a = R/2$. As before M is constrained to be real by hermiticity but can have either sign. Let us consider, for definiteness, that the RH component of the fermion is even and the LH component odd. Again the opposite parity assignments can be obtained by just replacing RH by LH and M by $-M$. Equation (6) is still valid with the derivative of the mass term being now

$$M'(y) = -\frac{M}{R} + M\pi[\delta(y) + \delta(y - \pi R)]. \tag{42}$$

The corresponding Schrödinger equation in the interior of the interval is

$$\left[-\partial_y^2+\frac{M}{R}+\frac{M^2}{R^2}(y-\pi R/2)^2\right]f_n^R=m_n^2f_n^R. \tag{43}$$

In this case the potential is even under reflections about the middle point $y=\pi R/2$. Then we can solve in the region $\pi R/2<y<\pi R$ and constrain the wave functions to be alternatively even and odd under this reflection. (Remember that we are solving for Ψ_R which is even under Z_2, and thus all wave functions are even at $y=\pi R$.) The boundary conditions read

$$f_{2n}^{R\,\prime}(\pi R/2)=f_{2n+1}^R(\pi R/2)=0,\quad f_n^{R\,\prime}(\pi R)=\frac{\pi M}{2}f_n^R(\pi R),\quad n=0,1,\ldots. \tag{44}$$

In order to solve the Schrödinger equation we first make the change of variables $t=\sqrt{\frac{M}{R}}(y-\pi R/2)$, which leads to the equation

$$[-\partial_t^2+(1-\lambda_n+t^2)]f_n^R=0, \tag{45}$$

where we have defined the dimensionless variable $\lambda_n\equiv\frac{m_n^2R}{M}$. We can now factorize the asymptotic form $f_n^R(y)=e^{-t^2/2}u_n(y)$, and make a further change of variables $z=t^2$. The resulting equation is a confluent hypergeometric equation

$$[z\partial_z^2+(\frac{1}{2}-z)\partial_z-\frac{1}{4}(2-\lambda_n)]u=0, \tag{46}$$

whose general solution can be written in terms of Kummer's functions $\mathcal{M}(a,b,z)$

$$u_n(z)=A\,\mathcal{M}\left(\frac{2-\lambda_n}{4},\frac{1}{2},z\right)+B\,z^{1/2}\,\mathcal{M}\left(1-\frac{\lambda_n}{4},\frac{3}{2},z\right). \tag{47}$$

Inverting the changes of variables and applying the corresponding boundary conditions we find

$$\begin{aligned} f_{2n}^R(y) &= A_{2n}\,e^{-\frac{M}{2R}(y-\pi R/2)^2}\,\mathcal{M}\left(\frac{2-\lambda_{2n}}{4},\frac{1}{2},\frac{M}{R}(y-\pi R/2)^2\right), & (48)\\ f_{2n+1}^R(y) &= A_{2n+1}\,(y+\pi R/2)\,e^{-\frac{M}{2R}(y-\pi R/2)^2}\,\mathcal{M}\left(1-\frac{\lambda_{2n+1}}{4},\frac{3}{2},\frac{M}{R}(y-\pi R/2)^2\right), & (49)\end{aligned}$$

where the masses $m_n^2=\frac{\lambda_nM}{R}$ are obtained solving the eigenvalue equations for λ_n:

$$\left(1-\frac{\lambda_{2n}}{2}\right)\mathcal{M}\left(\frac{3}{2}-\frac{\lambda_{2n}}{4},\frac{3}{2},\frac{\pi^2MR}{4}\right)-\mathcal{M}\left(\frac{1}{2}-\frac{\lambda_{2n}}{4},\frac{1}{2},\frac{\pi^2MR}{4}\right)=0, \tag{50}$$

for the even modes, and

$$\begin{aligned}&\left(1-\frac{\pi^2MR}{2}\right)\mathcal{M}\left(1-\frac{\lambda_{2n+1}}{4},\frac{3}{2},\frac{\pi^2MR}{4}\right)\\ &+\frac{\pi^2MR}{3}\left(1-\frac{\lambda_{2n+1}}{4}\right)\mathcal{M}\left(2-\frac{\lambda_{2n+1}}{4},\frac{5}{2},\frac{\pi^2MR}{4}\right) = 0,\end{aligned} \tag{51}$$

for the odd ones.

To gain some intuition we start considering the massless zero mode whose wave function can be written in terms of elementary functions. Using the property $\mathcal{M}(a,a,z) = e^z$ we find that $\lambda_0 = 0$ is a solution of the eigenvalue equation for the even modes. The corresponding zero mode has a gaussian profile

$$f_0^R(y) = A_0 \, e^{\frac{M}{2R}(y-\pi R/2)^2}, \tag{52}$$

being localized at $y = \pi R/2$ for $M < 0$ and at both fixed points with exponential suppression at the midpoint if $M > 0$. In the latter case the zero mode is multilocalized and we expect that the first KK mode be anomalously light. In Fig. 6 we plot the masses of the first few KK modes as a function of the slope of the five dimensional mass. The phenomenon of multilocalization and the corresponding light KK excitation is apparent for the appropriate M values. We also show in Fig. 7 the wave function profiles for the zero and first modes for the two M signs: $MR = -2$ with no multilocalization on the left; and $MR = 2$ with multilocalization on the right. There is a close resemblance of this spectrum with the one found in Section 2 for the case of a multikink mass term. This could have been anticipated from the shape of the potential with also two attractive delta functions at the orbifold fixed points.

As a final remark we would like to comment that in the flavour models with split fermions proposed in the literature the appropriate signs for the mass terms have been chosen (positive for even LH fields and negative for even RH ones) so as to have gaussian

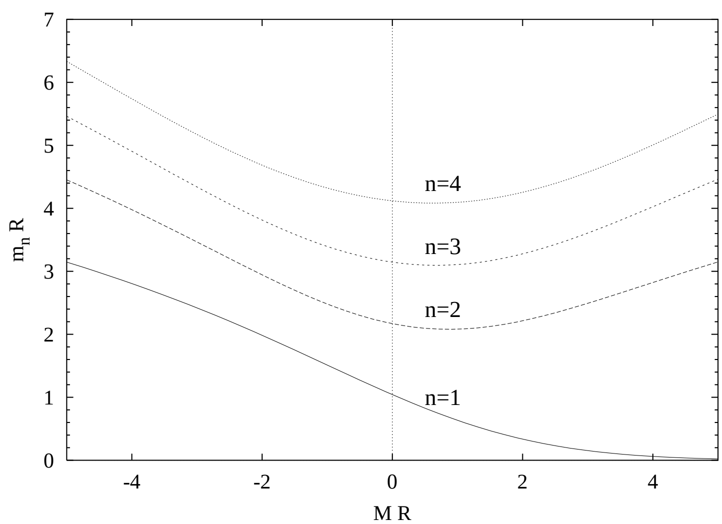

Figure 6: Values of the masses of the first KK modes for split fermions f_n as a function of the slope of the linear five dimensional mass.

localization around one single point and not multilocalization. However, we find no reason for a generic choice of sign excluding the possibility of multilocalization.

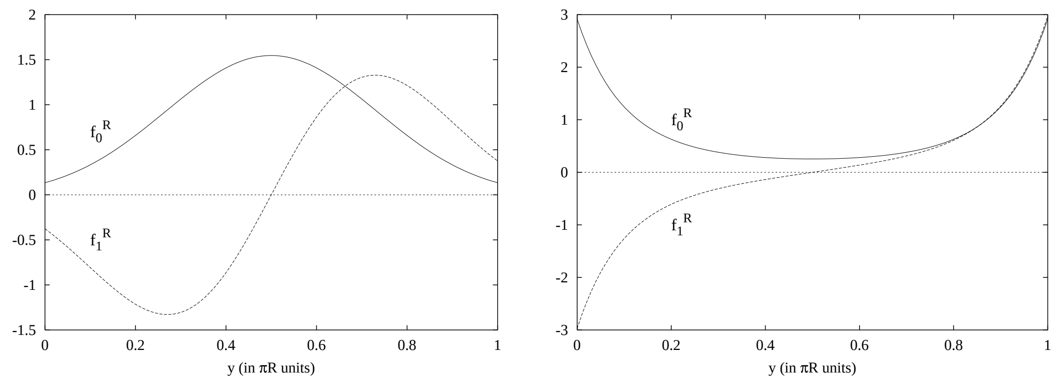

Figure 7: Profiles of the massless zero mode and the first KK excitation for the case of no multilocalization, $MR = -2$ (left), and multilocalization, $MR = 2$ (right).

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